Teaching problem solving in a Grade 5 mathematics class: A tweak and its impact on formal assessment

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Abstract: Participating in a research project, the researcher-teacher came to realize he often 'go over assigned work' as a way to teach problem solving. It involved giving students problems to try, and after some lapse of time, he would go over the assigned work. On reflection, such an approach offered his students a limited experience in problem solving. Students focused more on getting the steps to solve the problem and were seemingly able to do the given problems afterwards. There was very little *transfer* of learning to new or unfamiliar problems. To explore better practices, the researcher-teacher conceived of a way to tweak the usual approach. Gleaning from Polya (1957), viz., an Understand, Strategize, Execute and Reflect (USER) to explore a strategy to develop *better* problem solving skills is incorporated into the teaching of problem solving. The researchers conducted a teaching experiment. The USER approach is incorporated in one class of students over a period of the school year by the researcher-teacher. The other researcher conducted a pre- and post-test both in this class and another class for comparison purposes. Results from the pre- and post-tests show that the USER approach made some significant impact. A follow-up analysis on the year-end formal school examination showed statistically significant differences between the experimental and the contrast class. This paper reports on the results and the experiences of the researcher-teacher.
Keywords: Mathematical problem solving instruction, teacher research.

Introduction

Some evidence (Hedberg, Wong, Ho, Lioe, & Tiong, 2005) suggest that teachers often use 'going over assigned work' (GOAW) as a way to give mathematical problem-solving instructions. Typically it involves giving students problems to try, and after some lapse of time, the teacher would go over the assigned work, generally in one of the following three ways: (1) rework the problem, (2) focus on the procedures to solve the problem or (3) checking the answer quickly. Reworking the problem entails a thorough going over the problem, explaining what it is and showing students how it should be solved. Sometimes, the focus is more on the procedures needed to solve the given problems and at other times a quick check of the answer. There is very little need to go beyond the teaching and learning of steps to solve problems. Such an approach has its main focus on what Skemp (1976) termed as instrumental understanding. One consequence is students seem to be committing to memory the necessary rules and procedures involved in solving many word problems – an instrumental understanding that often does not translate well to other problems. On the surface, they have learned the steps to solve the problem, and are able to do the exercises with similar-looking problems shortly after. However they would likely face difficulties after some lapse of time, or when the problem presented does not look similar to the examples they were 'taught' earlier. Such an approach also has little room for solving problems through the four phases of problem solving recommended by Polya (1957), namely understand the question, devise a plan, carry out the plan and look back.

The researchers set out to investigate if a different approach by the teacher might result in students learning problem solving skills *differently*. Instead of giving problem solving instructions via the 'GOAW' way, an Understand, Strategize, Execute and Reflect (USER) approach based essentially on Polya's four phases of problem solving, is conceptualized and incorporated in the teacher's scheme of work. The focus is to develop an understanding of the process of solving problems instead of just approaching the task as a series of procedural steps to be followed by the students and for them to memorize and try to recall later. This paper reports on a study carried out in one primary six class in a Singapore school to find if such a shift in approach made any difference.

Method

The Study

Two classes of students were involved – one contrast and one experimental. The researcher-teacher adopted the USER approach to teach in the experimental class. The other class served as a contrast where another teacher taught in her *regular* way. Based on discussions, informal observations and a 50-minute lesson observation, the regular way generally entails a *traditional* expository approach that is *different* from the USER. There were 40 students in the experimental class and 39 in the contrast class.

The study involved the following: administering a pre-test for both classes, carrying out the USER approach in the experimental class, administering a post-test for both classes about ten weeks after the pretest. The researcher-teacher carried on with the USER approach for the remaining of the academic year and the students' results in the school's formal examination about four months after the post-test were analyzed.

The USER Approach

In the USER approach, a repertoire of mathematics problem-solving heuristics was first introduced to the students. These heuristics are referred to as the 'strategies'. Students are then taught *when* each strategy is to be used and *how* each strategy should be presented. Typically, some time is spent reading the question to the students, paraphrasing it and highlighting key words to them to ensure that they understand the problem. Following that, the students will be asked to suggest the strategy (or strategies) to use. To discourage students from rushing from reading the question to working out the solution, the teacher will pause here and ask the students why they have selected the strategy they have suggested. The objective is to show that the students have internalized the problem and given it some thought before they proceed to work out the solution. More importantly, this step allows the teacher to differentiate students who really know how to solve the problem from those who merely go through the motion by modeling after what they have copied from the board with little understanding.

The third step requires the student to work out the solution using the strategy that they have identified. To prevent the tendency of students to jump from reading the question to executing the strategy, the teacher provides a platform for a class level discussion to explore, plan and strategize. Very often, the students will identify and use more than one strategy. It is interesting to note that models or diagrams are often used merely to simplify the problem rather than to help them solve the problem.

The final step is Reflection. This is the most difficult step because not many students know exactly what their teachers mean when they hear, "Check your answers!" To many students, checking means ensuring that they have attempted all questions and that all answers are clearly shown in the space provided. Reflection is more than checking for computation errors or the occasional missing unit of measurement. Reflection requires students to explore if the problem could be solved using another strategy. It also requires them to ask themselves if the strategy they have employed to present their solution is the best. They can also look at their final answers and work backwards or ask if the answer they have arrived at fits logically back into the question (Polya, 1957).

The Instruments

The pre- and post-tests were paper-and-pencil tests, comprising five questions (see Appendix for a sample). To avoid bias, the researcher-teacher did not get to see the questions until after the post-test. Questions 1 and 2 of the tests are considered routine, and Questions 3, 4 and 5 non-routine. The questions in the post-test are essentially the same as those in the pre-test requiring similar conceptual understanding to solve, with slightly different wording and phrasing. The students took about 45 minutes to complete each of the tests. There was one student from each class who did not complete both the pre-test and post-test.

The school's formal exam was set, administered and graded by the school. It comprised three sections: A, consisting of multiple-choice questions, B, short-answer questions, and C, word problems.

The Scoring Procedure

The scoring procedure for both pre- and post-tests is adopted from Wong and Lim-Teo (2002), which is a modified version of a scale devised by Charles, Lester and O'Daffer (1987). There are three parts to each solution or response to a problem, namely, Understanding, Planning and Getting an Answer. Scores of 0, 1, or 2 are awarded for each part of the solution as shown in Table:

Table 1: *Analytic Scoring Scale (Charles, Lester, & O'Daffer, 1987)*

Understanding the problem	0	Complete misunderstanding of the problem
	1	Part of the problem misunderstood or misinterpreted
	2	Complete understanding of the problem
Planning a solution	0	No attempt, or totally inappropriate plan
	1	Partially correct plan based on part of the problem being interpreted correctly
	2	Plan could have led to a correct solution if implemented correctly
Getting an answer	0	No answer, or wrong answer based on an inappropriate plan
	1	Copying error; computational error; partial answer for a problem with multiple answers
	2	Correct answer

For Questions 1 and 2, which have parts, the same scale is applied to each part and the total score for that question is divided by the number of parts. This is to keep the weight of each question similar. The range of possible scores for each student is 0 to 30, with a maximum of six per question.

The section A of the exam paper consists of 15 multiple choice questions; the first five carry one mark each and the rest two marks each. An example of a one-mark question is "Mother bought 12 eggs and she broke some of them. Only a fraction of the eggs she bought were not broken. Which one of the following cannot be it?" The choices given were: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. An example of a two-mark question is: "There are red, blue and yellow beads in a box. There are 5 more blue beads than red ones. The number of yellow beads is twice that of the blue ones. If there are p yellow beads, how many red ones are there?" The choices of answers were: $2p + 5$, $2p – 5$, $\frac{p}{2}+5$, and $\frac{p}{2}-5$. Section B consists of 10 one-mark questions, mainly requiring short solutions of one or two steps. For example, "How many minutes are there in 2.3 hours?"

In section C of the exam paper, there are four two-mark questions, two three-mark questions, four four-mark questions and five five-mark questions, making a total of 15 word problems. An example of a word problem is: "Sally and Bala took a total of $10 to school one day. During recess, Sally spent $\frac{3}{4}$ of her money while Bala spent $\frac{1}{3}$ of his. They then had $5.50 left altogether. (a) How much money did Sally bring to school that day? (b) How much money did Bala have left?"

Findings and Analysis

The Pre- and Post-tests Findings

The means of the two classes in the pretest were compared. The following figure shows the plot of the mean scores with a 95% confidence interval:

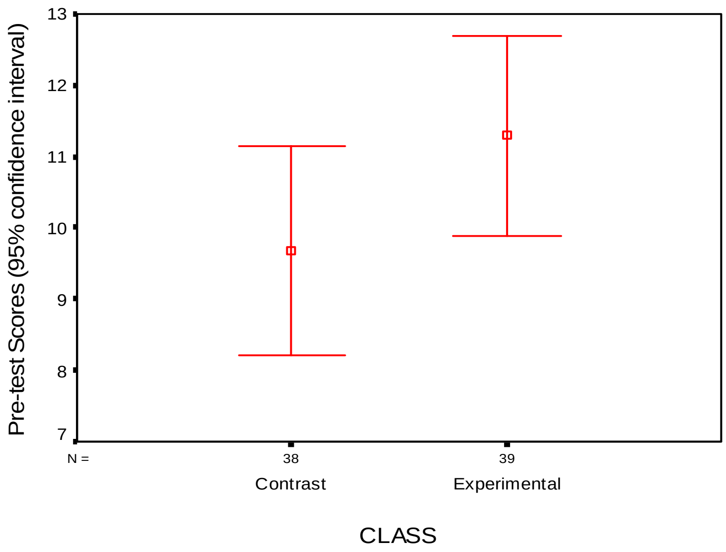

Figure 1: *Plot of pre-test mean scores of the contrast and experimental class with a 95% confidence interval*

At 95% confidence interval, the mean for the contrast class is between 8.22 and 11.14 while the mean for the experimental class is between 9.88 and 12.70. The independent samples t-test comparing the two means yielded $t(75) = 1.61$ and $p = 0.11$. Thus, although the mean of the experimental class is higher, there is no indication that it is significantly different from the contrast class.

Table 2*: Mean Performance Scores for Experimental and Contrast Classes*

	Experimental (n = 39)					Contrast (n = 38)				
	Pre-test		Post-test		Diff. in Post- & pretest means	Pre-test		Post-test		Diff. in Post- & pre-test means
	M	SD	M	SD		M	SD	M	SD	

Q1	2.74	1.82	3.92	2.01	1.18	2.12	1.88	2.75	1.98	0.63
Q2	3.68	1.73	4.45	1.24	0.77	2.83	1.42	3.43	1.53	0.60
Q3	2.72	1.49	3.10	1.35	0.38	2.53	1.37	2.11	1.52	-0.42
Q4	0.67	0.48	0.74	0.44	0.07	0.79	0.41	0.68	0.53	-0.11
Q5	1.49	2.16	3.79	2.02	2.30	1.58	2.06	2.18	2.24	0.60
All	11.29	4.35	16.02	3.36	4.73	9.84	4.39	11.15	4.40	1.31

Table 2 presents pre- and post-test means, standard deviations of the experimental and contrast classes, for each of the five questions and the overall in the tests. The maximum score is 30. Both classes made statistically significant improvement in their post-test scores over the pre-test. This could be due to the *learning* effect. However the improvements are much more substantive in the experimental class than the contrast. Considering the difference in mean scores between the classes, it is more marked in the post-test (a difference of 4.87 marks) than the pre-test (1.45 marks). For the contrast class, the mean score increased by 1.31 marks or about 0.30 SD while the experimental class increased by 4.73 marks or more than one SD. The standardized mean difference in the overall score (d = (mean$_E$ – mean$_C$)/SD) between the experimental and contrast class is 1.11. This means that the average student in the experimental class scores 1.11 times the SD of the contrast class above the average student in the contrast class. The effect size (calculated by $\frac{t}{\sqrt{n}}$) for the contrast class is approximately 0.36 which is small. For the experimental class, the effect size is about 1.22, an indication of a moderately strong effect size.

The following figure shows the plot of the mean scores for the post-test:

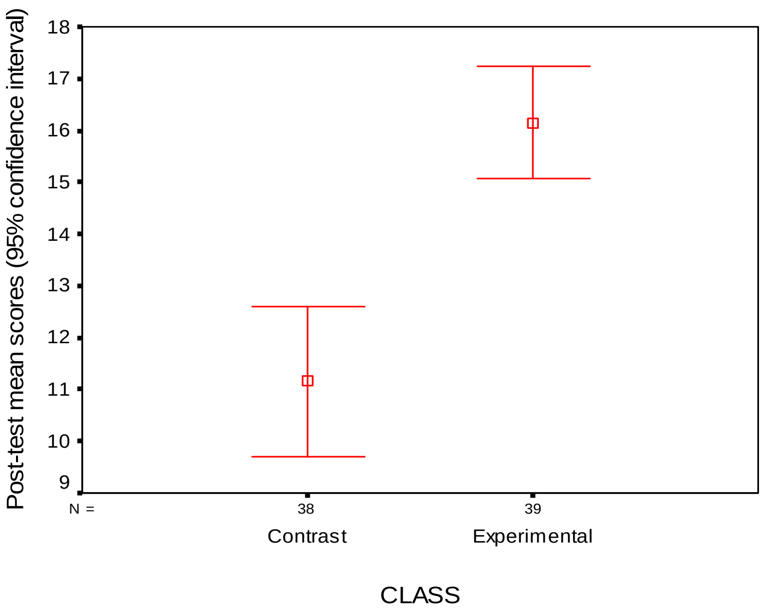

Figure 2: *Plot of post-test mean scores of the contrast and experimental class with a 95% confidence interval*

The confidence intervals of the mean scores did not overlap and the gap size of 2.4 marks is substantial. In addition comparing the post-test means of the two classes using the *t*-test, the value of $t(75) = 5.62$ and $p < 0.001$ indicate statistically significant difference between them.

Results from the school's formal examinations

Based on the initial results from the pre- and post-tests, the researcher-teacher continued with the USER approach. The school's formal examination was held about four months after the post-test. As noted, the researcher was not involved in the setting and the scoring. The following table shows the mean scores for the three sections and the overall mean scores:

Table 3: *Mean Formal Exam Scores for Experimental and Contrast Classes*

	Maximum Marks	Experimental (n = 40)		Contrast (n = 37)		Difference in Means
		Mean	SD	Mean	SD	
Section A	25	19.60	3.53	17.03	3.88	2.57
Section B	20	15.48	2.20	13.86	2.46	1.62
Section C	55	38.14	6.68	29.89	9.82	8.25
Overall	100	73.21	10.21	60.78	13.90	12.43

The difference in the mean scores between the contrast and the experimental class is about one SD, suggesting a statistically significant difference between the classes. The following plot of the mean scores of the contrast and experimental class with a 95% confidence interval illustrates:

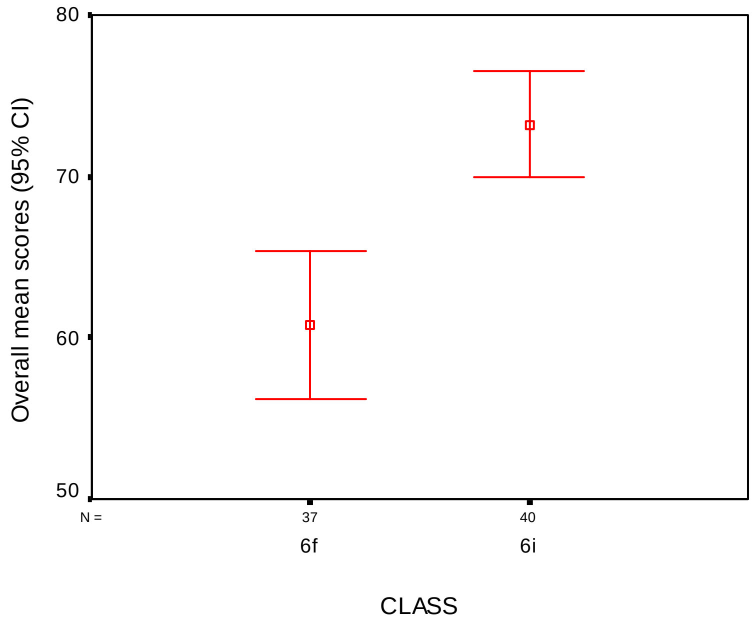

Figure 3: *Plot of examination mean scores of the contrast and experimental class with a 95% confidence interval*

In short, the findings and analysis of the pre- and post-tests results suggest that the USER approach made some impact helping students improved their problem solving skills compared with the contrast class, as measured by the holistic analytic marking scheme. The differences in their performances in the school's examinations also suggest that the USER approach made some impact beyond the pre- and post-tests.

The Researcher-Teacher's Experience

"The journey that led to this study began with my participation in a research project looking at problem solving in classrooms. My lessons were observed and video recorded by researchers (the co-authors of this paper) who reviewed and discussed what they saw with me.

They also conducted a workshop where the four phases of problem solving recommended by Polya (1957) were highlighted. A work attachment with the research team followed and I had the chance to learn about researching classroom practices firsthand. I realized as I observed other teachers the similarities and differences between what I saw and my own practices. This helped me become more reflective about what I do in class.

"One thing that I saw was teachers, including myself, tend to read the word problems quickly and emphasize only the 'execution' part of mathematical problem solving. We also tend to depend very much on one tried-and-tested strategy and 'teach' mainly through the answer-checking process and going over solutions as described in the earlier sections. On reflection this approach did not help students learn *more* actively. They were not given a chance to think as much as they should when solving problems. This was where the four phases of problem solving recommended by Polya came in. The idea was hatched that I would try out a new approach to incorporate the four phases into my teaching of mathematical problem solving. To help my students remember better I renamed them USER. Thus this small scale informal study came to be.

"As the results both in the post-test and the mid-year examination show, to some extent the USER approach had helped my students. From the experience I came to realize some attendant points that need to be highlighted. After going through the teaching experiment, I find the major differences between my previous approach and the USER are that with the USER approach, mathematical problem-solving skills are imparted to students in a *more structured manner* and that students are encouraged to *analyze* a mathematical problem systematically *before* they proceed to solve it. The USER approach may only be helpful to classes where the students already have a sound foundation in the basic mastery of mathematics skills. Teachers would need to build up their own repertoire of strategies first. I must add here that many teachers like me would need support to do this. As I build up my own repertoire I realize that there is not any one best time to introduce any particular strategy. The rule of thumb is to have many problem solving lessons and take it from there. I found working through a problem with one strategy and then on *reflection*, weaving in another strategy worked for me. I also found occasions where the *Draw a diagram/model* heuristic was used more for *understanding* the problem, and another heuristic employed to solve it – cases of using mixed strategies within one solution.

"It would take some time before students become familiar with the strategies, before they can strategize and execute them confidently. In the initial stages, students were reluctant to move beyond the *Draw a diagram/model* heuristic they were familiar with. It was only after some weeks that more of the other strategies (such as systematic listing, restating the problem) emerged from students' work and presentation. Further, teachers need to demonstrate each strategy from time to time and not show personal preferences for certain strategies they are more comfortable with so that the students would not have reservations about employing the one which they feel would help them present their solutions most elegantly. This way, students are not unduly limited by their teacher's preferences. Once I find my students are familiar with the USER approach, my lessons no longer need to be as teacher-centered as before. I began to 'go over' the solutions less, to tell the students *less*. There is more students' exploration of strategies; they become slightly more active in learning. I incorporated some group work and pair work to inject some variety to classroom setting, to promote some aspects of cooperative learning within the USER approach.

Conclusion

The researchers had set out to find if using the USER approach over a period of ten months or so helped improve students' problem solving skills. The shift in the teaching approach was from the more traditional 'answer-checking' or 'going over assigned work' way to the USER way entailing a building up of a repertoire of problem solving heuristics, more deliberation at *understanding* given problems, more explicit attempts at *strategizing* (i.e. planning) solutions and *reflecting* to check and look back for other ways of solving the problem. The statistically significant and moderately large improvement in the posttest of the experimental class suggests that the shift had made some impact as measured by the holistic analytic marking scheme. The difference in the mid-year examination scores between the contrast and the experimental classes further suggests that the USER approach made some impact in students' learning of MPS.

The study also highlighted areas where teachers would need support in if they were to adopt and adapt the USER approach in their classrooms. For a start teachers must build up their own repertoire of mathematical problem solving strategies. Workshops and short-term in-service courses might help. In addition, suitable material resources such as question banks with solutions should also be made available and easily accessible. To be effective in using the USER approach, some guidelines and initial hand-holding with peer review and reflection may be necessary. To support this, some collaboration with researchers or researcher-teachers is recommended. And for the researcher-teacher here, it brought both personal and professional development.

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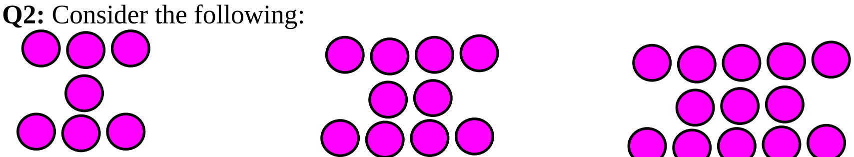
Appendix: Questions 1 to 4 of the 5 questions in the Post-test.

Q1: A group of five girls and four boys had a voucher to enjoy seven mini pizzas for free in a new restaurant. Cindy, one of the girls, suggested that the girls share four pizzas equally while the boys share three pizzas equally.

(i) Based on Cindy's suggestion, who would get the larger portion of pizza, a boy or a girl?

(ii) William, one of the boys, suggested that instead of following Cindy's idea, it would be better to share the seven pizzas equally. What is the difference between the two suggestions?

Q2: Consider the following:

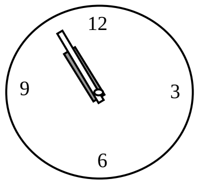


1st arrangement 2nd arrangement 3rd arrangement

(i) How many circles will be there in the 10th arrangement?
(ii) Write a rule or describe in words how to find the number of circles at every arrangement.
(iii) 99 circles are at the third row of one arrangement. Determine which arrangement it is.

Q3: Telephone company *X* charges its subscribers a monthly fee of $10.00 and 15 cents a minute. Telephone company *Y* charges a monthly fee of $12.00 and 12 cents a minute. Your uncle asks you which company's subscription is cheaper. How would you answer him?

Q4: Secret Agent X glanced at his watch and noticed that the hour and minute hands were *exactly* in line (see figure). He estimated he had slightly more than an hour to go before 12 o'clock to complete his mission. Find, as accurately as you can, how much time he had left?



Question 5

Belgian GP

Where	Spa-Francorchamps
Laps	44
Lap length	6.973km
Race distance	306.812km

Grid after final qualifying (top 10)

1	Jarno Trulli	(Ita)	Ren	1:56.232
2	Michael Schumacher	(Ger)	Fer	1:56.304
3	Fernando Alonso	(Esp)	Ren	1:56.686
4	David Coulthard (Gbr)	(Gbr)	McL	1:57.990
5	Giancarlo Fisichella (Ita)	(Ita)	Sau	1:58.040
6	Rubens Barrichello (Bra)	(Bra)	Fer	1:58.175
7	Mark Webber (Aus)	(Aus)	Jag	1:58.729
8	Felipe Massa (Bra)	(Bra)	Sau	1:59.008
9	Olivier Panis (Fra)	(Fra)	Toy	1:59.552
10	Kimi Raikkonen (Fin)	(Fin)	McL	1:59.635

Qualifying times in min:secs

Given the following information about the qualifying times the drivers took to complete **one** lap of the racing circuit:

Would the fastest qualifier, Trulli, overtake the slowest driver in the list, Raikkonen, during the race of 44 laps?