A View From A Singapore Mathematics Classroom: Good Practices and Teacher Development

HO, Siew Yin
National Institute of Education, Nanyang Technological University, Singapore
YEAP, Ban Har
National Institute of Education, Nanyang Technological University, Singapore

Abstract: Recent global reforms in the mathematics curriculum have propelled problem solving from an important component of school mathematics to a core competency in mathematics learning. The problem solving curriculum in Singapore acknowledges and reflects this emphasis. The current *Teach Less Learn More* initiative continues the Singapore Ministry of Education's vision to help every pupil develop competencies that are useful in a global, technological-based economy. Teachers, rather than providing all the explanations, are encouraged to help their pupils master the basics and to apply these basics to a wide range of situations. In order to contribute to the realization of this vision, teachers need to develop good practices in the mathematics classrooms. Good practices in the mathematics classroom can be characterized as providing pupils with opportunities to develop competencies and attitude that put them in good stead in the global, technological economy, and aiming to develop good thinking in students through enhancing pupils' thinking skills and thinking habits (Yeap, 2006). One part of the Think-Things-Through (T^3) Project aims to study how teachers can be engaged in developing innovative approaches and exemplify good practices. In traditional mathematics classrooms, teachers provide all the explanations and answers. It is argued that in good practices, teachers should be engaged in a wider range of roles other than the traditional one. How can and to what extent do teachers develop such good practices? Do teachers create environments and opportunities to engage their pupils in developing good thinking skills and thinking habits? Are teachers able to guide their pupils to develop key ideas in mathematics and extend their own knowledge? This paper offers a lesson as an illustration of good practice in the mathematics classroom. A teacher-innovator model for good practices, which comprises of four stages, is proposed to investigate the extent of teacher development.

Keywords: problem solving, thinking skills, thinking habits, good practices, teacher development

Introduction

Recent global reforms in the mathematics curriculum have propelled problem solving to a core competency in mathematics learning. The problem solving curriculum in Singapore acknowledges and reflects this emphasis.

Problem-solving was made the central focus of the Singapore mathematics curriculum in 1992. The implementation of the vision of Thinking Schools Learning Nation (TSLN) in 1997 aimed to promote the teaching of thinking in the Singapore mathematics classroom. A thinking person has the ability to consider and assess the context of a given situation. In other words, he or she is capable of making sense of the given situation.

In 2001, the revision of the Singapore mathematics curriculum encourages the explicit teaching of thinking skills. As a critical part of the Thinking Schools, Learning Nation (TSLN) initiative, the Innovation and Enterprise (I & E) initiative encourages teachers to help students develop a set of positive habits of mind. Habits of minds are dispositions of intelligent persons in response to problems (Costa & Kallick, 2000a). Habits of mind help students "see that the responsibility for thinking is theirs" (Costa & Kallick, 2000b, p. 2).

In 2005, schools are encouraged to "teach less" so that the students can "learn more"; teaching would be focused on developing understanding, questioning and critical thinking. (Lee, 2004; MOE, 2005b, 2005c). Thus the current *Teach Less Learn More* initiative continues the Singapore Ministry of Education's vision to help every pupil develop competencies that are useful in a global, technological-based economy.

In 2007, the Singapore mathematics curriculum is revised to emphasize the recent initiatives. Specifically, teachers are encouraged to de-emphasize paper-and-pencil computation and to emphasize mental computation and skills like visualization.

With each revision of the Singapore mathematics curriculum, the focus continues on developing a thinking person in our students. A thinking person is capable of considering and assessing the context of a given situation. In other words, he or she is capable of making sense of the given situation. Overall, the education system and the mathematics curriculum aim to help students develop competencies that are useful in the global technological knowledge-based economy.

The Think-Things-Through (T^3 Project)

One part of the Think-Things-Through (T^3) Project aims to study how teachers can be engaged in developing innovative approaches and exemplify good practices. Briefly, the T^3 Project provides worksheets for teachers to use in their mathematics lessons. Teachers are encouraged to read notes for the teachers provided by the research team and to discuss with each other before the implementation. The two sources for teacher development are worksheets and the discussion. The worksheets are available at http://math.nie.edu.sg/T3

Teacher Practice and Good Practices

In early studies on teachers' practices, "teachers' practice" was mostly defined as "acts" or "actions" or "behaviours" of the teachers. This way of defining the word " teachers' practice" has evolved over the years with many researchers suggesting various definitions. Simon and Tzur (1999) considered "teacher' practice" as a conglomeration of "everything [that] teachers [do to contribute] to their teaching (planning, assessing, interacting with students)…everything teachers think about, know, and believe about what they do" (pp. 253-254) and also the teachers' skills, values, intuition and feelings about their teaching (p. 254). According to Boaler (2003), teacher' practices are "the recurrent activities and norms that develop in classrooms over time, in which teachers and students engage" (p. 3).

Many studies have attempted to analyze classroom practices (e.g., Schoenfeld, Minstrell, & van Zee, 2000; Stephens, 2006, Zimmerlin, & Nelson, 2000) and to describe what "good practices" or "best practices" in teaching are (e.g., CIMT, 2004; Burghes (Ed.), 2004; Inprasitha, Loipha, & Silanoi, 2006; Isoda, 2006; Lim, 2006; Marsigit, 2006; Pérez, 2006; Stevenson & Stigler, 1992; Stigler & Hiebert , 1999; Takahashi, Watanabe, & Yoshida, 2006; Ulep, 2006; Vui, 2006). In particular, Isoda (2006) identified good questioning technique by the teacher to be a form of good practice. Takahashi, Watanabe, and Yoshida (2006) believed that such good practices should promote the development of all five interwoven and independent strands of mathematical proficiency namely, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Ellerton (as cited in Lim, 2006) proposed that the best practice will involve "listening to the voices of the children" (p. 205). Lim (2006) outlined six characteristics of good practices:

(i) student centered activities that encourage conceptual understanding
(ii) related to students' daily life experiences
(iii) that the students understand what is being taught and can apply what they have learned to solve problems
(iv) good planning of student activities
(v) active participation of students in fun and meaningful activities
(vi) use of teaching aids that enhance student understanding

(p. 210)

Inprasitha, Loipha, & Silanoi (2006) emphasized the need to satisfy certain criteria in their description of what good practice constitutes. The criteria are, alignment with their country's Education Act and their new curriculum, support of teacher as a researcher, and creation of teacher networks.

Indeed "good practices" is not the same all over the world and is dependent on the goal of instruction and of the curriculum of each country. Ellerton (2003) recommended that models of such good practices "need to be shared" (as cited in Lim, 2006).

The Think-Things-Through (T^3) Project proposed that good practices in the mathematics classroom be characterized as the following:

(1) Good practices provide students opportunities to develop competencies and attitude that put them in good stead in the global, technological economy.
(2) Good practices aims to develop good thinking by enhancing students' thinking skills and thinking habit.
(3) Good practices instill among students a belief that they are able to extend their own knowledge.
(4) Good practices engage students in the learning process.
(5) Good practice is effective as in all students develop key ideas in mathematics.

(Yeap, 2006, p. 234)

Bowers, Cobb & McClain (1999) pointed out that classroom mathematical practices will evolve through the discussion of problems and solutions between the teacher and the students. In traditional mathematics classrooms, teachers provide all the explanations and answers. In good practices teachers, active agents of education innovation, should be engaged in a wider range of roles other than the traditional one. What are some of the good practices that contribute to the realization of the vision to help students develop competencies that are useful in the global technological knowledge-based economy? How can and to what extent do teachers develop such good practices? Do teachers create environments and opportunities to engage their pupils in developing good thinking skills and thinking habits? Are teachers able to guide their pupils to develop key ideas in mathematics and extend their own knowledge?

This paper attempts to describe a lesson designed by the teacher in a Singapore school. Previously, she had conducted lessons prepared by the research team. The lesson that she designed and conducted had features that are similar to the lessons by the research team.

An Illustration of Good Practice in a mathematics classroom

The lesson is based on one of the Think-Things-Through Project Primary Four worksheets, *Cubes*. The teacher has been involved in the project since the beginning of the project. The class size is 38 and the students are of mixed-ability.

The teacher created her own worksheet based on the *Cubes* worksheet, and used it for the lesson that was video-recorded. The teacher created two main tasks, Task 1 and Task 2 in her worksheet.

In Task 1, students were asked to find the number of square faces of two, three, four and five cubes. They were also required to find the number of square faces of a solid figure formed by two, three, four and five cubes. Task 2 seeks to investigate the relationship between the number of square faces formed and the position of the cubes in the solid figure.

The ensuing section presents description of teacher behaviour in two snapshots of the lesson illustrating good practice through the Think-Things-Through Project's proposed criteria (Yeap, 2006, p. 234) mentioned in the previous section. The reader is asked to refer to Appendices A & B for the two snapshots of the lesson.

(1) Good practices engage students in the learning process:

The students were actively involved in solving the tasks. They had made their own concrete materials, i.e., cubes, in a previous lesson and worked on the tasks using the cubes. The students worked in pairs, and certain pairs of students had the opportunity to share and justify their solutions to the tasks and their thinking strategies with their peers. Sufficient time was also given to the students for exploration.

(2) Good practices aims to develop good thinking by enhancing students' thinking skills and thinking habit:

The teacher used a repertoire of verbal strategies to engage the students in the lesson. Through these verbal strategies, the teacher coached the students on the usage of thinking skills, thus making the teaching and learning of thinking skills conscious and explicit (see Lee & Yeap 2002; Lee, 2006; Yeap, 2006, June). Thinking skills are skills that can be used in a thinking process, such as classifying, comparing, sequencing, analyzing parts and wholes, identifying patterns and relationships, induction, deduction and spatial visualization (MOE, 2005a). Coaching is a "tool that acts as a catalyst to encourage learning and improve performance based on increased self-awareness and self-accountability (Ng, 2004, p. 1).

Yeap (2005) suggested three methods to help students develop good habits of mind. Firstly, teacher can model good habits of mind to their students by thinking aloud during problem solving in class. Habits of minds are dispositions of intelligent persons in response to problems (Costa & Kallick, 2000a). Teachers can also coach their students to show such habits of mind, that is, teachers can ask questions such as "Is there another way to look at this". Thirdly, teachers can create opportunities for students to show habits of mind by providing them such situations in the classroom. Refering to Appendix B, the teacher asked the class whether there was another way to form a solid using 4 cubes.

The teacher used the strategy of revoicing the students' utterances. For example (in Appendix A), the teacher asked the students for the number of square faces of the solid formed using three cubes. The students answered, "Fourteen!". The teacher revoiced the students' utterances by saying "Fourteen". Revoicing is a particular kind of re-utterance of one's contribution by another participant in a discussion (O'Connor & Michaels, 1996).

O'Connor & Michaels (as cited in O'Connor & Michaels, 1996) pointed out that revoicing by teachers in classroom conversations creates participant frameworks that facilitates students to align themselves with the given classroom tasks and their socialization to roles and identities in intellectual discourse. Through revoicing, the teacher is also able to construct her understanding of her students' understanding (考查学习, 发现他知道什么) and thus align her understanding with her students (与学生取得协调) (see Fang, 1971, pp. 299-301)

Initiation-Response-Evaluation (IRE) is a frequent strategy used by the teacher in her lesson. IRE is a technique frequently used by classroom teachers. In IRE, a question is initiated by the teacher, followed by a response from a student, and followed in turn by evaluation of response by the teacher.

Throughout the lesson, the teacher pressed for justifications, explanations, and meaning through questioning, comments, feedback. This helped in scaffolding the students' thinking and reasoning skills.

(3) Good practice is effective as in all students develop key ideas in mathematics:

The tasks given were accessible to every student. Each student in the class was able to achieve some degree of success in the tasks. The use of the cubes made the tasks even more accessible. In view of recent reforms that encourage a shift from teaching heuristics to developing habits of mind to accompany the use of heuristics, Yeap (2005) highlighted the move towards students' acquisition of "big math ideas" skills such as number sense, patterns and relationships, visualization and modeling. The students were given practice in one of the four important "big math ideas", in particular their spatial visualization skills. In various parts of the lesson, the teacher made spatial visualization explicit (refer to Appendix B).

During the lesson, the teacher summarized what the students had found out and learnt twice. In the first summary, the teacher elicited from the students that the difference in number of square faces in each solid depended on how one positioned the cubes in the solid. In the second summary, the teacher asked the students whether they would find out the number of faces of a solid made up from a large number of cubes by counting the number of square faces on the solid one at a time. She recapped the strategy they had used in the lesson: subtract the number of sets of covered faces from total number of square faces of the cubes making up the solid. The teacher did not emphasize the computation aspect. Instead, a key idea in mathematics, making generalization, was given emphasis.

(4) Good practices provide students with opportunities to develop competencies and attitude that put them in good stead in the global, technological economy:

The teacher created the environment and gave her students the opportunity to be engaged in developing good thinking skills and habits of mind. She also modeled and coached her students in metacognitive skills; that is, being aware of, able to control, monitor and self-regulate one's thinking processes, in particular the selection and use of problem-solving strategies. The teacher encouraged her students to communicate their strategies with each other by asking pairs of students to present in front of their classmates. Communication is an important part of the mathematical process as it helps students to "develop their own understanding of mathematics and sharpen their mathematical thinking" (MOE, 2005a). By getting students to work in pairs and also giving students opportunities to share and present their solutions and ideas to their peers, the teacher not only engaged her students in cultivating confidence in using mathematics and maintaining interest in the mathematical tasks, but also in developing interpersonal skills. Such competencies and positive attitude will equip and put students in good stead in the global, technological economy.

Conclusion and future recommendations

This paper attempted to answer the following questions:

1. What are some of the good practices that contribute to the realization of the vision to help students develop competencies that are useful in the global technological knowledge-based economy?
2. How can and to what extent do teachers develop such good practices?
3. Do teachers create environments and opportunities to engage their pupils in developing good thinking skills and thinking habits?
4. Are teachers able to guide their pupils to develop key ideas in mathematics?

A teacher development model, the Teacher-Innovator model, for good practices to investigate the extent of teacher development is proposed. The Teacher-Innovator model, comprising of four stages, is described in Table 1.

Table 1: *Teacher-Innovator model*

Level	Description
Level 0	Teachers are indifferent to the stimulus provided. Such stimuli include the lessons designed by the research team and any discussion the teacher may have with his or her colleague. Other than the lessons designed by the research team, no change is detected in other lessons.
Level 1	Teachers respond to the stimulus provided in a superficial manner. For example, the lessons they design are identical to those designed by the research team. Any modifications are superficial.
Level 2	Teachers respond to the stimulus provided in s structural manner. For example, the lessons they design are structural modifications of the lessons designed by the research team.
Level 3	Teachers become innovators of good practices. They no longer modify lessons they receive from the research team. Instead, they design their

	own lessons.

Under what conditions is each level in the Teacher-Innovator Model achieved? What are the beliefs and goals of the teacher at each level? What is the level of teacher knowledge of content and pedagogy like in each level? What are the lesson plan and lesson image (see Schoenfeld, 2000) of the teacher in each level like? These and others are questions for further investigation.

References

Boaler, J. (2003). Studying and capturing the complexity of practice: The case of the dance of agency. In N. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th PME International Conference, 1*, 3-16.

Bowers, J., Cobb, P., & McClain, K. (1999). The evolution of mathematical practices: A case study. *Cognition and Instruction, 17* (1), 25-64.

Burghes, D. (Ed.). (2004). *International* project on mathematical attainment report *[Monograph 4]. Series of International Monographs on Mathematics Teaching Worldwide.* UK: University of Exeter, Centre for Innovation in Mathematics Teaching (CMIT).

Centre for Innovation in Mathematics Teaching (CMIT) (204). *Examples of good practice in primary mathematics teaching from around the world: IPMA international project on mathematical attainment* [DVD]. UK: University of Exeter, CMIT.

Costa, A. L., & Kallick, B. (2000a). *Discovering and exploring habits of mind.* Alexandra, VA: Association for Supervision and Curriculum Development.

Costa, A. L., & Kallick, B. (2000b). *Habits of mind: Activating & engaging.* Alexandra, VA: Association for Supervision and Curriculum Development.

Fang, B. L. (*方炳林*) (1971). *普通教学案法 (The Method of Classroom Teaching)*.Taiwan: 三民书局有限公司(San Ming Press).

Inprasitha, M., Suladda, L., & Silanoi, L. (2006). Development of effective lesson plan through lesson study approach: A Thai experience. T*sukuba Journal of Education Study in Mathematics, 25,* 237-245.

Isoda, M. (2006). Reflecting on good practices via VTR based on a VTR of Mr. Tanaka's lesson 'How many blocks?'. T*sukuba Journal of Education Study in Mathematics, 25,* 181-195.

Lee, H. L. (2004, August). *National Day Rally Speech.* Speech delivered at the National Day Rally, Singapore.

Lee, N. H. (2006, June). *The nature of mathematical reasoning & how to teach it.* Keynote lecture given at the Mathematics Teachers Conference, National Institute of Education, Nanyang Technological University, Singapore.

Lee, N. H. & Yeap, B. H. (2002). Thinking in the secondary mathematics classroom: Issues and lessons. In A. Chang & Y. M. Cheah (Eds.). *Teachers' handbook on teaching thinking across disciplines* (pp. 71 - 82). Singapore: Prentice Hall.

Lim, C. S. (2006). In search of good practice and innovation in mathematics teaching and learning: A Malaysian perspective. T*sukuba Journal of Education Study in Mathematics, 25*, 205-219.

Ministry of Education (MOE) (2005a). *Mathematics Syllabus (Primary).* Singapore: Ministry of Education.

Ministry of Education (MOE) (2005b). *Nurturing every child: Flexibility & diversity.* Singapore: Ministry of Education.

Ministry of Education (MOE) (2005c). *Touching hearts, engaging minds: Preparing our learners for life.* Singapore: Ministry of Education.

Ng, P. T. (2004). *Grow me!: Coaching for schools.* Singapore: Pearson Education South Asia.

O'Connor, M. C., & Michaels, S. (1996). Shifting participant frameworks: orchestrating thinking practices in group discussion. In D. Hicks (Ed.), *Discourse, Learning And Schooling.* Cambridge: Cambridge University Press.

Pérez, G. G. (2006). Beginning the study of the additive field. T*sukuba Journal of Education Study in Mathematics, 25*, 257-271.

Schoenfeld, A. H. (2000). Models of the teaching process. *The Journal of Mathematical Behaviour, 18* (3), 243-261.

Schoenfeld, A. H., Minstrell, J., & van Zee E. (2000). The detailed analysis of an established teacher's non-traditional lesson. *Journal of Mathematical Behaviour , 18* (3), 281-325.

Stephens, M. (2006). Strategies for addition and subtraction of whole numbers extended to number sentences involving fractions and decimals. T*sukuba Journal of Education Study in Mathematics, 25*, 247-255.

Simon, M. A., & Tzur, R. (1999). Explicating the Teacher's Perspective from the Researchers' Perspectives: Generating accounts of mathematics teachers' practice. *Journal for Research in Mathematics Education, 30* (3), 252-264.

Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap.* NY: Simon & Schuster.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap.* NY: The Free Press..

Takahashi, A., Watanabe, T., & Yoshida, M. (2006). Developing good mathematics teaching practice through lesson study: A U.S. perspective. T*sukuba Journal of Education Study in Mathematics, 25*, 197-204.

Ulep, S. A. (2006). Good mathematics teaching practices-In the making: A Philippine experience. T*sukuba Journal of Education Study in Mathematics, 25*, 289-297.

Yeap, B. H. (2005). Helping every student to develop the ability in mathematical problem solving: The Singapore experience. P*roceeding of Symposium on Mathematics Education: Mathematical Problem Solving,* 3-12.

Yeap, B. H. (2006, June). *The nature of mathematical reasoning & how to teach it.* Keynote lecture given at the Mathematics Teachers Conference, National Institute of Education, Nanyang Technological University, Singapore.

Yeap, B. H. (2006). Good practices in mathematics teaching and teacher development. T*sukuba Journal of Education Study in Mathematics*, *25*, 233-236.

Zimmerlin, D., & Nelson, M. (2000). The detailed analysis of a beginning teacher carrying out a traditional lesson. *Journal of Mathematical Behaviour , 18* (3), 263-279.

Appendix A

A snapshot from the lesson:

Finding the number of square faces on the solid formed using three cubes (in Task 1 of the worksheet)

The students had already found the number of square faces of a solid that is made up from three cubes. Jonathan and partner's strategy to find the number of faces of a solid was to subtract the number of covered faces in the solid from the total number of faces of each cube making up the solid.

Teacher: How about the number of square faces on the solid figure?

Students: Fourteen!

Teacher (repeats after children): Fourteen. Why is that fourteen?

Jonathan: Because er...four sides is covered.

Teacher: Okay, Jonathan, partner come out. Come on here.

Teacher: Okay, can show the class? Can explain to the class?

[Jonathan walks to the front of the classroom with three cubes in his hands. Partner follows.]

Teacher: The rest can you please listen.

Teacher (instructing Jonathan and partner to move closer to their classmates' desks): ...closer please.

Jonathan: Four of the lines make...[Partner points to the intersections]

Partner: Are covered.

Jonathan: Are covered...make...

Partner (uses finger to point around the solid): Outside only...then because the outside...the inside...

[Both Jonathan and partner moved unconsciously away from their classmates as they were explaining. Teacher walks towards students and signals them to move closer to their classmates' desks.]

Jonathan (excitedly): covered...

Teacher: [to Jonathan] Slowly, slowly...[to the class] all listen.

Jonathan: Two of the faces put together [brings two cubes face-to-face with each other], you can't see them. Another two [brings the other two cubes face-to-face] so you just count…

Teacher: So we must take away two?

Jonathan and partner (disagreeing): Four!! [Partner laughs]

Teacher: Ah, look ah..three cubes but you must take away two sets of covered sides. Am I right?

Jonathan: Ya, two times two equals four. So eighteen minus four equals...fourteen

Teacher: Okay, eighteen minus four equals fourteen. [goes to whiteboard to write 18-4=14]

[Jonathan and partner walk back to their seats with their solid.]

Teacher: Okay, but this four will be two sets of covered sides [draws arrow down from the number "4" on white board, and writes the words "2 sets"]. Am I right? Okay?

Appendix B

Another snapshot from the lesson:
Finding the number of ways to form a solid using 4 cubes (in Task 2 of the worksheet)

A pair of students had earlier shared with the class their way of forming a solid using four cubes. The teacher then asked whether there was another way to form a solid using 4 cubes. A child, Hady, put up his hand and the teacher called him to share his solution with the class.

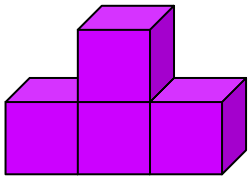
Teacher:…is there one more way?

[Hady puts up his hand.]

Teacher: Hady!

[Hady gets up from his seat and moves to the front of the classroom.]

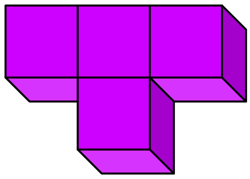
Hady (showing his solid to his classmates) : One way...



Teacher: (walks toward Hady and helps him hold the solid) Ya...this is one way...ya...er...however, this one is going to have how many sets of cov...er covered faces?

Some students: Three.

Teacher: Three ya...Alright, if I am going..if I'm going to turn this way (orientates the solid)...opposite like that, is it the same?

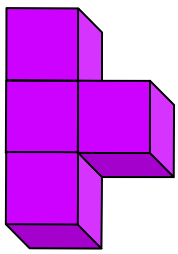


Students: Yes.

Teacher (repeats her question): Is it the same? [Lets go of the solid. Hady now holding onto the solid.]

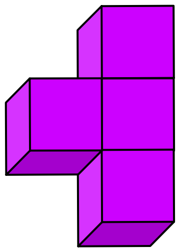
Students: Yes.

Teacher: Ya, still the same ah...How about this way? [orientates the solid]



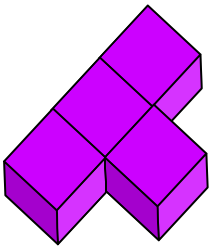
Students: The same.

Teacher: Same right? The other way…this way. [orientates the solid]



Students: Yes!

Teacher: Same ah..even if you tilt this [orientates solid], it is the same, right?



Students: Yes!

Teacher: This one has the same structure. Do you see that? Okay? Okay, very good.