MPCK Variables Valued by Schools' Mathematics Department Heads

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Abstract: Mathematics Pedagogical Content Knowledge (MPCK) affects classroom practices which in turn affects learning by pupils. While there are many desirable practices which demonstrate strong MPCK, teachers' adoption and use of these practices in part depend on the value placed on them by the school management. This paper presents the quantitative and qualitative findings of a survey as well as insights from interviews of primary schools' Mathematics department heads on their perceptions of the importance of various teachers' practices in contributing towards effective mathematics teaching and learning. The study shows that the department heads value practices which contribute towards conceptual learning and pupil motivation to learn which go beyond achievement in performance tests.
Keywords: mathematics pedagogical content knowledge, teacher practices in mathematics, perceptions of primary schools' mathematics department heads, effective mathematics teaching

Introduction

The Mathematics Pedagogical Content Knowledge (MPCK) research project at the National Institute of Education in Singapore was initiated to study the development of MPCK in beginning teachers. It seems almost obvious that teacher characteristics with respect to their pedagogical content knowledge would have impact on student learning. However, it is far more difficult to establish empirically the relationship between teachers' knowledge and their students' mathematical achievement. Mewborn (2003) summarized that connections between student achievement and teacher characteristics have not been conclusive and that the relationship is nonlinear and threshold effects seem to occur. Nevertheless, case study evidence suggests that teachers' mathematics pedagogical content knowledge (MPCK) has a strong influence on children's learning outcomes (Shulman, 1986). In mathematics**,** Ball (2000) stressed how the depth of teachers' understanding of MPCK is a major determinant of teachers' choice of examples, explanations, exercises, items, and reactions to children's work.

The need for identification of different constructs as applied to Singapore's primary mathematics curriculum resulted in the development of an instrument to measure MPCK. This instrument was described in the first paper of this symposium. While teacher

performance on the instrument provides some indication of their MPCK level at a theoretical level, it is, nevertheless, the teachers' actual practice in classrooms which ultimately affects pupil learning. An important part of the research was to determine what the researchers term "MPCK-in-action" outcomes as observed being practised by teachers when teaching mathematics and to ascertain the relative importance of different practices in contributing towards effective pupil learning.

A large scale study of lessons in Singapore showed that students in mathematics classes are mainly engaged in activities focusing on factual/rote knowledge and procedural computation. Yeo & Zhu (2005) analysed data collected from 18 primary five classes and noted that answer checking, individual seatwork and monologue teaching occupied 39%, 32% and 13% respectively of all the phases in the mathematics lessons. However, Luke (2005) postulates that while the approach of questioning merely to elicit specific pre-determined "correct" answers may seem less learner-centred compared to an approach of creating knowledge through discussion with the learners, it could well be an effective approach for establishing the correct concept and use of terminology in specific subjects such as mathematics.

While particular teacher actions or practices may or may not occur in any particular mathematics lesson, the reporting officers of the mathematics teachers would have professional perceptions of the relative contributions made towards pupil learning by these practices. Such perceptions, which reflect school cultures in terms of what the management values among various teaching practices, will affect the teachers' use of the different practices in their mathematics lessons. Knowing the practices valued by Heads of Mathematics Departments in schools will provide data from the practitioners' perspectives and also help researchers to understand the rationale behind teachers' choice of practices. This paper will report on the findings on the perceptions of Heads of Mathematics departments in schools.

Research Method

The MPCK-in-action practices are identified through (a) an analysis of video-tapes of some mathematics lessons conducted by beginning teachers and (b) the researchers' experience in observing student-teachers in their school practicum. Brainstorming, discussion and verifying with the video observations resulted in a list of 35 observable practices in mathematics lessons which were considered effective for learning. Reversing the process, the researchers re-visited the framework consisting of important MPCK constructs as set out earlier in the project and on which the MPCK instrument mentioned in the introduction was based. These constructs are (i) Generic teaching skills, (ii) Multiple representations of concepts, (iii) Content knowledge with deep understanding, (iv)

Understanding of the cognitive demands of mathematical learning tasks and (v) Ability to identify learner difficulties and misconceptions. These constructs could be exemplified by ten categories of actions or practices in mathematics lessons and it was found that the 35 observable practices could be located under these ten categories. The categorization is given in Table 1 below. A questionnaire consisting of the list of 35 observable teacher practices was then designed, requiring mathematics Heads of Departments (HODs) to rate on a scale of 0 to 10 the importance of each practice in contributing towards effective mathematics learning in their pupils.

Table 1: *Survey items according to categories*

General Categories	Observable Mathematics Teachers' Practices
Sequencing of Activities	1. Sequences learning activities logically 2. Structures examples/tasks from simple to complex 3. Adopts the concrete-pictorial-abstract approach where applicable
Choice of Activities	4. Identifies and selects the most suitable learning activity to teach a certain topic 5. Designs/modifies learning activities to match pupils' learning needs 6. Uses a variety of learning activities to develop the given concept
Connections between topics and between concepts	7. Builds on pupils' prior knowledge to teach new knowledge 8. Relates/applies concepts to the real world context 9. Provides opportunity to integrate topics/concepts learnt 10. Makes links between topics/concepts
Balance between concept development and mathematical procedures	11. Consciously emphasizes the underlying reasons/explanations for the given mathematical procedure 12. Focuses the pupils on the essential steps and necessary conditions in procedures 13. Emphasises computational speed and accuracy 14. Allows pupils to explore alternative procedures in solutions 15. Provides pupils the opportunity to identify and rectify errors in presentation of solutions 16. Places due emphasis on conceptual understanding

Explanation	17. Explains mathematical terms accurately 18. Explanations are appropriate to the learners' level 19. Explains clearly and concisely 20. Uses appropriate range of examples 21. Uses non-examples to enhance pupils' understanding of concepts 22. Uses multiple modes of representations for developing concepts or establishing procedures 23. Provides counter-examples for the concept/procedure

Table 1 (contd)

Role model in demonstrating mathematical processes	24. Models exemplary mathematical behaviour (e.g. Being logical and systematic in presentation, using mathematical instruments correctly)
General Categories	Observable Mathematics Teachers' Practices
Mathematical communication	25. Uses correct mathematical terms and language 26. Makes provision for pupils' mathematical communication
Mathematics curriculum	27. Displays knowledge of Singapore mathematics curricular emphasis and syllabus requirements
Questioning techniques	28. Monitors pupils' understanding through appropriate questions 29. Uses structured questioning to facilitate development of concepts 30. Uses structured questioning to establish mathematical procedures 31. Asks questions to stimulate mathematical thinking processes e.g. comparing, classifying, generalizing, deducing etc.
Responding to students	32. Detects pupils' errors/misconceptions 33. Analyses the cause of pupils' errors/misconceptions 34. Takes appropriate actions to rectify errors and/or correct misconceptions 35. Adopts alternative explanation/representation when pupils encounter difficulty in learning concepts/procedures

Besides the items to be rated, the HODs were given opportunities to clarify and elaborate their views through three open response questions. Specifically, the questions provided opportunities for them (i) to explain their ratings, (ii) to identify two most important practices which could include practices not listed among the 35 items and (iii) to explain how these valued practices would influence pupils' learning outcomes.

These questionnaires were mailed to about 180 primary school mathematics HODs and 81 returns were received. After the quantitative analysis, a series of one-one interviews were conducted. A representative sample of 10 HODs was chosen from the pool of respondents for the one-one interviews to seek clarifications on their rating of the items and obtaining a fuller picture of HOD's thinking on practices in mathematics teaching. This sample comprised HODs from different school types in the four different zones, varying number of years of teaching experience, academic and professional qualifications.

The interview started, as an introduction, by eliciting information on how these key personnel appraise their teachers' mathematics teaching performance. Questions were asked on the mathematics practices which they valued, particularly those have given a rating of 8 or more and those they valued the least. They were then asked to elaborate on how these practices have impact on pupils' learning in terms of performance and attitude.

Key findings and discussion

Quantitative analysis of Survey Items

From the demographic data of the 81 HODs who responded, the distribution according to years of mathematics teaching experience is given in Table 2.

Table 2: *Distribution of HODs across Experience in Teaching Mathematics*

Years of Mathematics Teaching Experience	Percentage of the 81 respondents
Up to 5 years	11.1 %
6 – 10 years	22.2 %
11 – 20 years	23.5 %
More than 20 years	43.2 %

About 25% have a degree as their highest academic qualification though not necessarily a degree in Science or Mathematics and 56% had taken a full-time professional diploma specially tailored for preparing experienced teachers to become department heads.

The Cronbach alpha for the whole instrument is 0.97. This high reliability coefficient of the 35-item instrument coupled with the fact that each item is equally weighted based on the first principal component analysis confirms that each item contributed to an important aspect of the positive impact of good teacher practices.

A one-way ANNOVA results on the mean of all items was employed to verify whether the responses were affected by the respondents' number of years of teaching mathematics, number of years as HOD, their highest academic qualification and professional qualifications. None of them yielded any significant differences at .05 level.

On a scale of 0 to 10 rating, 10 being the most important, the mean rating score for each item fell between 7.46 and 9.35 and the standard deviation for each item between 0.59 and 3.52. Table 3 below shows the three items with the lowest and highest ratings. The three lowest rated terms were the only three items where the mean rating fell below 8.00.

Table 3: *Three Lowest Rated and Three Highest Rated Items of Mathematics Teacher's Practices*

Item No.	Mathematics Teacher's Practices	Mean	S.D.
Lowest Rated Items			
21*	Uses non-examples to enhance pupils' understanding of concepts	7.46	3.52
13	Emphasises computational speed and accuracy	7.49	2.68
23	Provides counter examples for the concept/procedure	7.96	2.04
Highest Rated Items			
18	Explanations are appropriate to the learners' level	9.28	0.90
34	Takes appropriate actions to rectify errors and/or correct misconceptions	9.30	0.59
19	Explains clearly and concisely	9.35	0.79

*Note: *3 respondents rated this item 0 and 2 respondents left it unrated*

The lowest rated item 21 was due to three respondents giving it a zero rating which also accounts for the high standard deviation. However, the same three HODs gave high ratings of 8 and above to item 23 which was to provide counter-examples for concept or procedure. Nonetheless, item 23 was also a comparatively low-rated practice. It can be argued that the HODs may not have clearly understood how non-examples can be used to enhance concepts since two HODs actually left the item unrated. As item 23's use of counter-examples was for procedures as well as concepts, it may have been rated more highly due to "procedures". It is quite common for teachers to show how pupils should not carry out procedures but uncommon to show non-examples of concepts or counter-examples of statements.

The high ratings of items 18, 19 and 34 indicate the importance of clear and precise explanations and knowing how to rectify errors and misconceptions appropriately. These practices are very strongly emphasized to Singapore teachers as essential to promote clear understanding by pupils and strong performance in achievement tests.

Interestingly, the results shows that the HODs' profile and background do not play any crucial role in identifying similar good teachers' practices in contributing to effective mathematics learning.

Results from Open-Ended Questions

The responses to the open-ended questions in the survey showed the practices which they valued contributed towards two overarching goals: (a) conceptual understanding by pupils and (b) the affective aspects of motivating pupils through helping them understand and enjoy

mathematics. What came through in the open responses was that HODs were very concerned with true mathematical learning beyond mere computational speed and accuracy which also explained the relatively lower rating of item 13. They valued the practices which contributed towards these goals and these practices can be grouped into five themes as follows.

The first is that effective mathematics teachers should adopt the concrete-pictorial-abstract (CPA) approach wherever possible in the teaching of mathematics to ensure pupils have conceptual understanding. This is not surprising as the CPA approach is the approach advocated in the Singapore mathematics syllabus since the 1980s. The HODs also saw the practice of using manipulatives and activities as being ways of engaging learners.

The second theme substantiates the ratings of two of the top three items, i.e. the ability to present materials and explain clearly. Several HODs commented that "*The teachers must be strong in mathematics pedagogies and must ensure that their explanation is clear, . . ., presented in a logical and systematic way*." Choice of activities, importance of sequencing the activities with clear explanations are perceived as essential in Mathematics pedagogy.

The third theme is the recognition that mathematics is a hierarchical subject, therefore there is a need to review prior knowledge and teach from familiar to unfamiliar. As another HOD shared, *"… because many topics are thought in isolation, pupils are not aware of the links between them. Therefore it is crucial that the teachers help to make links between the topics or concepts so pupils know why the topics/concepts are important."*

The fourth theme is the importance of relating mathematics to the real world context so that pupils could see the meaning of the mathematics they are learning. This is how one of the HODs put it, *" I think it is essential that pupils are able to relate what they have learnt to the real world because then they will see meaning in what they are learning."*

The last theme is related to the second highest rated item and the ability to diagnose errors and remediate errors is also another important mathematics teaching practice. As one HOD puts it, "*Teachers must have adequate intervention skills when they diagnose a particular learning disability or lack of mastery of a mathematics skill."*

For the last question in the questionnaire, the HODs did not refer to the list of teaching practices to explain in depth how particular practices would influence pupils' mathematics learning outcomes. Instead they explained why the two main goals as given above were important for pupil learning to take place. With regard to conceptual understanding, one

HOD wrote: "*Understanding concepts is crucial for Mathematics learning. No meaningful learning takes place if pupils are not aware of underlying concep*ts *within each topic"*. Another wrote: "*Experiential learning helps pupils understand and 'see' concepts on their own"*. From what they have stated, HODs value the conceptual understanding and they expect teachers are able to ensure that pupils gain conceptual understanding through different learning experiences. The value of experiential learning have several spin-offs such as pupils take ownership of their learning, pupils have deeper understanding of the concepts and have longer retention of what they have learnt.

Qualitative analysis:

In the one-to-one interviews, HODs shared with the researchers their experiences in their recent appraisals of mathematics teachers, giving examples of their Mathematics teachers who displayed those valued practices and their profiles. They also provided more information on what they value in pupils' mathematics learning in relation to Singapore Mathematics curriculum framework. The analysis of the HODs' interviews surfaces some emerging trends which will be discussed below.

Appraisal Procedures

Every school carries out lesson observation as part of the appraisal process. Most teachers are observed once a year while "weaker" teachers were observed twice. However, for some schools, only new teachers are observed. Most HODs regard pre-conference sessions as developmental for the teachers and during their observations, 80% used a generic observation checklist while others give the observation report to teachers during the post conference. The post conference sessions are generally considered as feedback sessions by the reporting officer (HOD) and teachers are asked to reflect on their practices in the observed lessons either orally or in written form. This is usually done on the very day itself otherwise it is done within the week of observation. Several schools use the Ministry of Education's Enhanced Performance Management System (EPMS) form and rate their teachers using the performance indicators which place great emphasis on generic pedagogy and class management.

HODs were also careful to include practices outside lesson time in their appraisal. Other supplementary modes of appraisal include checking the quality of worksheets designed by teachers; whether they were appropriately customised for the class level or ability. In assessing their quality of marking workbooks or worksheets, they are rated in terms of accuracy, comments and follow-up actions.

Practices that are valued highly by the HODs

The HODs noted certain general competencies and attributes which they valued. They firmly believed that without good class management, literally, no teaching could be carried out, much less effective teaching leading to effective learning. They also believed that without passion, teachers will not have the motivation and commitment to exercise these desirable practices on a sustained basis. This aspect was not covered in our questionnaire because the project was focusing on particular practices applicable to mathematics only and not on teaching in general.

The HODs in the interviews stressed that pedagogy was important although they differed or were vague in what they meant by effective pedagogy. In one case, the HOD who believed strongly in the importance of pedagogy said that during the feedback session, she would provide some guidelines on what her teachers should concentrate on while acknowledging that it could appear prescriptive. She said, "*….pedagogy actually, I focus a lot on that. Because ultimately, that is something that is very important. That is something that we can control……if you want to see improvement, then it is best if there are some guidelines.*"

On particular practices, the interviews provided a rich pool of their perceptions of good mathematics teaching practices which were not very different from the open-ended responses in the survey. The interviews thus confirmed with elaborations the reasons for the valued practices under the five themes as given earlier.

In one primary school with a higher proportion of pupils with above average ability , the Mathematics HOD values the practices like "*emphasizing conceptual understanding*", "*explains mathematical terms accurately*", "*explanations appropriate to learners' level*" and "*takes appropriate actions to rectify pupils' errors*". Another valued practice is that of teacher modeling alternative approaches in solving the same problem. In another school with relatively high ability pupils, the HOD valued the concrete-pictorial-abstract approach because she believes that pupils learn differently. Another HOD echoed the same view when he said "*The C-P-A approach is something that is a very, very natural progression for pupils . ..",* believing that it is essential to provide the scaffolding due "*to their developmental level*". Due to the abstract and hierarchical nature of mathematics, another HOD cherishes the practice, "*to sequence learning activities logically*" and building on prior knowledge for effective learning.

As discussed earlier, practices such as "*Detect pupil's misconception/erro*r", "*Analyze pupil's error*", "*Take appropriate actions to rectify error and/or correct misconception*" are valued by HODs for this is one way to help teachers realise why children do not understand their lessons. Another HOD also valued the same practices for a slightly different reason.

This HOD thinks that unless the teacher probes into the thinking of the pupils, it is futile for teacher to conduct remedial lessons, teaching the same thing over again.

"*Relating to real life situation*" is another highly valued practice. One of the HODs recognised that this is necessary due to the abstract nature of mathematics. Another HOD felt that these practices of "*Apply concepts to the real world problem*" and "*Provides opportunity to integrate topics/concepts learnt*" serve to motivate pupils to learn mathematics. This intrinsic motivation will enhance the pupils' performance in tests and examinations.

Several HODs highlighted questioning techniques as being *very important* as modeling the questioning process assists pupils to ask themselves of the same questions in future, an essential element in developing independent learning. One particular HOD concurred with the importance of questioning techniques but she interpreted the process to mean questioning how things can be done differently, which is in line with the project *Habits of Minds* that her school is currently implementing.

The practice "*Consciously emphasizes the underlying reasons/explanations for the given mathematical procedure*" is also valued by one HOD because it brings out the beauty of mathematics. However, he acknowledged that it can be difficult to implement and that the actual implementation depends very much on the nature of pupils that the teacher is teaching. For the same practice, another HOD saw it from a different perspective; she felt that this practice helps weaker pupils to understand the steps in the problem-solving process.

For school with a higher proportion of weaker pupils, the HOD views "*structuring simple to complex*" as very important because it help pupils understand the subject. She also regarded "*explains mathematical terms accurately*" as vital because it helps pupils to understand word problems. "*Knowing how to use the mathematical tools"* is also very important as her pupils need a lot of guidance in this area.

One HOD also pointed out that strong content is a necessary condition for strong PCK: "*…without content knowledge, I think the rest cannot be achieved. Even though the teacher may be able to deliver, but there's no communication. There's no transfer of knowledge …*"

The above discussion suggests that the different reasons for HODs valuing the good practices are very much dependent on two main factors: (a) their pupils' characteristics and abilities and how these practices can best assist and motivate pupils in their learning, and (b) the HODs' own views of the nature of mathematics. These prized practices, as aptly pointed out by an HOD, act as an "*indirect indication of the teacher's possession of pedagogy content knowledge"* in Mathematics.

There are also external influences on HODs' perceptions of good practices such as cluster initiatives and professional development of the HODs themselves. For instance, two cluster workshop themes; *co-operative learning and using questioning techniques to develop habits of mind* appear to have impact on their beliefs of good Mathematics pedagogy. Others quoted that takeaways from National Institute of Education's course for preparation of HODs and other in-service courses have impacted their perceptions of what good MPCK-in-action meant.

Practices that are less valued by HODs

Some anecdotal explanations of their lower ratings provided the researchers new insights into their perceptions. While some of the practices are generally highly valued, there were some qualifications, especially concerning the weaker pupils.

HODs' perceptions of teaching practices like "*provision of non-example*" and the "*use of multiple representations*" as ***not so relevant*** was in the context of their attempts to tailor teaching to their weaker pupils' learning styles. Moreover, time constraint is also another factor due to the need to complete syllabus and slower learning pace of these pupils. The HODs felt that providing "*one or two examples*" is sufficient, "otherwise *having too many may even confuse these pupils*". Also, "e*mphasising computation speed and accuracy*" is also considered as less important because lower ability pupils need time to carry out their computations correctly and speed should take lower priority.

The practice of relating "*Mathematics to the real world*" and "*providing opportunities pupils to identify or provide alternative solutions*" are not favoured by some HODs as they remarked that "*concept building is more important, focus on essentials and get the fundamentals right*". Interestingly, another HOD reckons that these sets of practices require some skills on the part of the teachers and felt that these practices are not as important if his teachers are not ready to use them well.

According to one HOD, the practice to "*focus the pupils on the essential steps and necessary conditions in procedures*" is deemed not as important. In the climate of developing pupils to be innovative and enterprising, such a practice may not promote thinking as it appears to be prescriptive in nature. Yet, another HOD thinks that it is not important to sequence activities as some activities stand alone. "*Using counter examples and non-examples*" as part of the teaching pedagogy is less valued as an HOD points out it is fairly difficult to find counter examples to illustrate the teaching of a concept.

The C-P-A approach which was valued highly by a number of HODs was deemed not as important by one HOD as she felt that this approach is more relevant in the teaching of lower primary levels but not so at the upper primary levels. In addition, while she regarded as important that teachers should "*display knowledge of Singapore curricular emphasis and syllabus requirements*", it may be unrealistic to expect this from her teachers.

The information from the interviews seems to suggest what HODs regard as less important are attributed to a few reasons such as the nature or characteristics of the pupils, the relevance of the practices at the different grade levels and the readiness of teachers concerned.

Practices that have an effect on pupil mathematical development

These HODs have a broad view and a diverse blend of beliefs on what constitutes pupil mathematical development. A number of them identified mathematical development to encompass areas like (i) test performance, (ii) ability to appreciate how mathematics is related to daily life, (iii) stretching pupils cognitively and (iv) having a positive attitude towards the subject.

One HOD is of the view that that mathematics is a hierarchical subject and therefore, a spiral approach that builds on pupils' existing knowledge and skills is effective in enhancing mathematical development. Two HODs whose pupils are of below-average ability also emphasised building on prior knowledge for the same reason stated above. Three other HODs advocate a hands-on approach through learning activities to help pupils enjoy mathematics. Though the approach may not translate into better mathematics performance, they are confident that it will have impact on these pupils' performance in the long run or affect positively their dispositions towards Mathematics.

One theme that emerged was that the teachers are important role models in the mathematical development of pupils. Teachers' modeling of how to overcome errors and misconceptions is also found to be important as it will help pupils to persevere in the event that they make mistakes or are stuck in problem solving. Teachers' modeling of exemplary mathematical behaviour like being logical and systematic is also important because this will help pupils to be systematic and logical in their presentation.

Another theme was the need to go beyond mathematical concepts and skills specified in the syllabus or textbooks. The HODs would want teachers to teach of thinking skills and processes, as well as make mathematics meaningful by moving beyond textbook questions. They felt that this would serve to make learning holistic and also change pupils' view of mathematics learning.

HODs' evaluation of the practices that impact on pupils' mathematical development depends to a large extent, on the pupils' characteristics, how the pupils under their charge learn best and also how the HODs themselves defined mathematical development.

Concluding Remarks

The survey and interview findings show that in general, school mathematics heads of departments value practices which promote understanding of concepts and which enhance their pupils' attitude towards mathematics rather than those which only concentrate on developing skills in procedures. The evaluation of MPCK practices by HODs is primarily determined by the characteristics of pupils in each school, their knowledge of their Mathematics teachers' readiness and the HODs' personal MPCK and experiences.

While the HODs strongly felt that these MPCK-in-action practices were desirable for effective teaching and learning of Mathematics, there was some concern with the ability of their teachers to carry out these practices well. In their annual appraisal of Mathematics teachers, HODs used generic observation checklists rather than a Mathematics-specific instrument. The list of 35 items which this study has used could be a starting point for discussions within particular schools to determine the relevant and desirable mathematics teaching practices for their teachers. With a more subject-specific checklist, HODs can then help their teachers to do profiling with respect to their practices in teaching mathematics, thereby guiding them to more focused professional development where necessary.

References

Ball, D. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning how to teach. *Journal of Teacher Education, 51*, pp 214 – 247.

Luke, A. (2005). Making New Asian Pedagogies: Classroom interaction in Singapore Classrooms. Keynote Address. Redesigning Pedagogy: research, policy, practice - International Conference on Education, Singapore.

Mewborn, D.S. (2003). Teaching, teachers' knowledge and their professional development. In J. Kilpatrick, W.G. Martin & D. Schifter (Eds.), *A research companion to the principles and standards for school mathematics*, pp 45 – 52. Reston, VA: NCTM.

Shulman, L. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 15*, pp 4 – 14.

Yeo, S.M. & Zhy, Y. (2005). Higher-order thinking in Singapore Mathematics classrooms. Unpublished conference paper. Redesigning Pedagogy: research, policy, practice - International Conference on Education, Singapore.

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